

# Complete Intersecting Non-Extreme $p$ -Branes

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## Abstract

We give general intersecting brane solutions without assuming any restriction on the metric in supergravity coupled to a dilaton and antisymmetric tensor fields in arbitrary dimensions  $D$ . The result is a general class of intersecting brane solutions which interpolate the non-extreme solutions of type 1 and 2. We also discuss the relation of our solutions to the known single brane solution.

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Understanding classical solutions of supergravities in eleven and ten dimensions is an important subject in the current particle physics. These are the low-energy effective theories of string and M theories. An important class of solutions in such theories are the extended objects called branes [1]-[4], which have played significant role in our study of nonperturbative effects in strings and field theories realized on the branes. In particular non-extreme solutions give rise to non-extreme black holes and thus are very important in studying the properties of realistic black holes. Various supersymmetric and non-extreme solutions, and their intersections have been studied so far [5]-[18].

It has been known that there are two possible ways to construct non-extreme solutions, classified as type 1 and 2 in ref. [12]. Type 1 has the metric

$$ds^2 = e^{2A} dx_{p+1}^2 + e^{2B} (dr^2 + r^2 d\Omega_{\tilde{d}+1}^2), \quad (1)$$

where the dimension of the space-time is given as  $D = p + \tilde{d} + 3$  and there is no restriction on the functions  $A$  and  $B$  except that they are functions of  $r$  only. The usual extreme solutions are obtained under the condition [5]

$$(p+1)A + \tilde{d}B = 0, \quad (2)$$

which can be understood as ‘no-force’ or BPS condition. By type 1 non-extreme solutions, we mean that the restriction (2) is removed.

The metric for type 2 solutions is taken as

$$ds^2 = e^{2A} (-f dt^2 + dx_p^2) + e^{2B} (f^{-1} dr^2 + r^2 d\Omega_{\tilde{d}+1}^2), \quad (3)$$

with the restriction (2). Here the function  $f$  gives the non-extreme extension.

There have been many works on these two kinds of non-extreme solutions separately [3]-[19], but to the best of our knowledge neither clarification of the connection of these solutions nor attempt at interpolating these two classes of solutions have been made. In view of the importance of both these solutions, it is interesting to examine if there are more general solutions that include both classes of solutions and hence interpolate these in the particular limits of the parameters. The purpose of this paper is to show that this is indeed possible by deriving complete intersecting brane solutions without the restriction (2). We also discuss their relations to other known solutions.

The method adopted here is a simple generalization of that developed by one of the present authors some time ago [15] for the type 2 solutions. There the field equations were solved with a simplifying ansatz which generalizes the condition (2). What we show here is that it is in fact possible to solve the field equations without this ansatz, and the result is a very general class of solutions that involve additional integration constants, and their appropriate choices give both the solutions of type 1 and 2.

Let us start with the general action for gravity coupled to a dilaton  $\phi$  and  $m$  different  $n_A$ -form field strengths:

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^m \frac{1}{2n_A!} e^{a_A\phi} F_{n_A}^2 \right]. \quad (4)$$

This action describes the bosonic part of  $D = 11$  or  $D = 10$  supergravities; we simply drop  $\phi$  and put  $a_A = 0$  and  $n_A = 4$  for  $D = 11$ , whereas we set  $a_A = -1$  for the NS-NS 3-form and  $a_A = \frac{1}{2}(5 - n_A)$  for forms coming from the R-R sector.<sup>3</sup> To describe more general supergravities in lower dimensions, we should include several scalars as in ref. [3], but for simplicity we disregard this complication in this paper.

From the action (4), one derives the field equations

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \sum_A \frac{1}{2n_A!} e^{a_A\phi} \left[ n_A (F_{n_A}^2)_{\mu\nu} - \frac{n_A - 1}{D - 2} F_{n_A}^2 g_{\mu\nu} \right], \\ \square\phi &= \sum_A \frac{a_A}{2n_A!} e^{a_A\phi} F_{n_A}^2, \\ \partial_{\mu_1} (\sqrt{-g} e^{a_A\phi} F^{\mu_1 \dots \mu_{n_A}}) &= 0, \\ \partial_{[\mu} F_{\mu_1 \dots \mu_{n_A}]} &= 0. \end{aligned} \quad (5)$$

The last equations are the Bianchi identities.

We take the following metric for our system:

$$ds_D^2 = -e^{2u_0} f dt^2 + \sum_{\alpha=1}^p e^{2u_\alpha} dy_\alpha^2 + e^{2B} \left[ f^{-1} dr^2 + r^2 d\Omega_{\tilde{d}+1}^2 \right], \quad (6)$$

where  $D = p + \tilde{d} + 3$ , the coordinates  $y_\alpha$ , ( $\alpha = 1, \dots, p$ ) parametrize the  $p$ -dimensional compact directions and the remaining coordinates of the  $D$ -dimensional spacetime are the

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<sup>3</sup>There may be Chern-Simons terms in the action, but they are irrelevant in our following solutions.

radius  $r$  and the angular coordinates on a  $(\tilde{d}+1)$ -dimensional unit sphere, whose metric is  $d\Omega_{\tilde{d}+1}^2$ . Since we are interested in static spherically-symmetric solutions, all the functions appearing in the metric as well as dilaton  $\phi$  are assumed to depend only on the radius  $r$  of the transverse dimensions.

If the resulting metric has null isometry, say, in the direction  $y_1$ , we can incorporate the boost charge by a well-defined step [20, 8]. Since this is quite straightforward, we simply concentrate on the diagonal metric (6).

For background field strengths, we take the most general ones consistent with the field equations and Bianchi identities. The background for an electrically charged  $q_A$ -brane is given by

$$F_{0\alpha_1\cdots\alpha_{q_A}r} = \epsilon_{\alpha_1\cdots\alpha_{q_A}} E', \quad (n_A = q_A + 2), \quad (7)$$

where  $\alpha_1, \dots, \alpha_{q_A}$  stand for the compact dimensions. Here and in what follows, a prime denotes a derivative with respect to  $r$ .

The magnetic case is given by

$$F^{\alpha_{q_A+1}\cdots\alpha_p a_1\cdots a_{\tilde{d}+1}} = \frac{1}{\sqrt{-g}} e^{-a_A \phi} \epsilon^{\alpha_{q_A+1}\cdots\alpha_p a_1\cdots a_{\tilde{d}+1} r} \tilde{E}', \quad (n_A = D - q_A - 2), \quad (8)$$

where  $a_1, \dots, a_{\tilde{d}+1}$  denote the angular coordinates of the  $(\tilde{d}+1)$ -sphere. The functions  $E$  and  $\tilde{E}$  are again assumed to depend only on  $r$ .

The electric background (7) trivially satisfies the Bianchi identities but the field equations are nontrivial. On the other hand, the field equations are trivial but the Bianchi identities are nontrivial for the magnetic background (8).

In the above metric (6), the function  $f$  is introduced to describe the type 2 non-extreme solutions. Here we also define nonvanishing function

$$\sum_{\alpha=0}^p u_\alpha + \tilde{d}B = \ln X, \quad (9)$$

to describe type 2 non-extreme extension. In ref. [15], the field equations (5) were solved with the simplifying ansatz that the combination (9) vanishes. Although this was the only assumption there, we show here that it is not mandatory and that the field equations (5) can be solved in a wider context without such ansatz.<sup>4</sup>

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<sup>4</sup>This deformation was also considered in ref. [17] for a single brane and in [18] for intersecting branes in pp-wave spacetime. There the function  $f(r)$  in the metric was put to 1.

In order to solve the field equations (5), we need the Ricci tensors for our metric (6). The non-zero components are

$$\begin{aligned}
R_{00} &= e^{2(u_0-B)} f^2 \left[ \left( u_0 + \frac{1}{2} \ln f \right)'' + \left( \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) \left( u_0 + \frac{1}{2} \ln f \right)' \right], \\
R_{\alpha\beta} &= -e^{2(u_\alpha-B)} f \left[ u_\alpha'' + \left( \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) u_\alpha' \right] \delta_{\alpha\beta}, \quad (\alpha, \beta = 1, \dots, p), \\
R_{rr} &= - \left( B + \frac{1}{2} \ln f + \ln X \right)'' - \sum_{\alpha=0}^p (u_\alpha')^2 - \tilde{d} (B')^2 + \left( \frac{X'}{X} - \frac{\tilde{d}+1}{r} \right) B' \\
&\quad - \frac{f'}{2f} \left( 2u_0' + \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right), \\
R_{ab} &= -f \left[ (B + \ln r)'' + \left( \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) (B + \ln r)' \right] g_{ab} + \frac{\tilde{d}}{r^2} g_{ab}, \quad (10)
\end{aligned}$$

where  $g_{ab}$  is the metric for  $(\tilde{d}+1)$ -sphere of radius  $r$ .

For both cases of electric (7) and magnetic (8) backgrounds, we find that the field equations (5) are cast into

$$\left( u_0 + \frac{1}{2} \ln f \right)'' + \left( \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) \left( u_0 + \frac{1}{2} \ln f \right)' = \frac{1}{f} \sum_A \frac{D - q_A - 3}{2(D-2)} S_A (E_A')^2, \quad (11)$$

$$u_\alpha'' + \left( \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) u_\alpha' = \frac{1}{f} \sum_A \frac{\delta_A^{(\alpha)}}{2(D-2)} S_A (E_A')^2, \quad (\alpha = 1, \dots, p), \quad (12)$$

$$\begin{aligned}
&\left( B + \frac{1}{2} \ln f + \ln X \right)'' + \sum_{\alpha=0}^p (u_\alpha')^2 + \tilde{d} (B')^2 - \left( \frac{X'}{X} - \frac{\tilde{d}+1}{r} \right) B' \\
&+ \frac{f'}{2f} \left( 2u_0' + \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) = -\frac{1}{2} (\phi')^2 + \frac{1}{f} \sum_A \frac{D - q_A - 3}{2(D-2)} S_A (E_A')^2, \quad (13)
\end{aligned}$$

$$f \left[ (B + \ln r)'' + \left( \frac{f'}{f} + \frac{X'}{X} + \frac{\tilde{d}+1}{r} \right) (B + \ln r)' \right] - \frac{\tilde{d}}{r^2} = - \sum_A \frac{q_A + 1}{2(D-2)} S_A (E_A')^2, \quad (14)$$

$$r^{-(\tilde{d}+1)} X^{-1} \left( r^{\tilde{d}+1} f X \phi' \right)' = - \sum_A \frac{\epsilon_A q_A}{2} S_A (E_A')^2, \quad (15)$$

$$\left( r^{\tilde{d}+1} X S_A E_A' \right)' = 0, \quad (16)$$

where  $A$  denotes the kinds of  $q_A$ -branes and we have defined

$$S_A \equiv \exp \left( \epsilon_A a_A \phi - 2 \sum_{\alpha \in q_A} u_\alpha \right), \quad (17)$$

and

$$\delta_A^{(\alpha)} = \begin{cases} D - q_A - 3 \\ -(q_A + 1) \end{cases} \quad \text{for } \begin{cases} y_\alpha \text{ belonging to } q_A\text{-brane and } \alpha = 0 \\ \text{otherwise} \end{cases}, \quad (18)$$

and  $\epsilon_A = +1(-1)$  corresponds to electric (magnetic) backgrounds. For magnetic case we have dropped the tilde from  $E_A(r)$ . Equations (11), (12), (13) and (14) are the  $00, \alpha\alpha, rr$  and  $ab$  (angular coordinates) components of the Einstein equation in eq. (5), respectively. The last one is the field equation for the field strengths of the electric backgrounds and/or Bianchi identity for the magnetic ones.

From eq. (16), one finds

$$r^{\tilde{d}+1} X S_A E_A' = c_A, \quad (19)$$

where  $c_A$  is a constant. With the help of eq. (19), eq. (11) can be rewritten as

$$\left[ r^{\tilde{d}+1} f X \left( u_0 + \frac{1}{2} \ln f \right) \right]' = \sum_A \frac{D - q_A - 3}{2(D - 2)} c_A E_A', \quad (20)$$

which can be integrated to give

$$f X \left( u_0 + \frac{1}{2} \ln f \right)' = \sum_A \frac{D - q_A - 3}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_0 \tilde{d}}{r^{\tilde{d}+1}}, \quad (21)$$

where  $c_0$  is an integration constant. Similarly, we find that eqs. (12) and (15) give

$$\begin{aligned} f X u_\alpha' &= \sum_A \frac{\delta_A^{(\alpha)}}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\alpha \tilde{d}}{r^{\tilde{d}+1}}, \quad (\alpha = 1, \dots, p), \\ f X \phi' &= - \sum_A \frac{\epsilon_A a_A}{2} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\phi \tilde{d}}{r^{\tilde{d}+1}}, \end{aligned} \quad (22)$$

where  $c_\alpha$  ( $\alpha = 1, \dots, p$ ) and  $c_\phi$  are again integration constants. We find from eq. (14) the result

$$f X (B + \ln r)' - \frac{\tilde{d}}{r^{\tilde{d}+1}} \int r^{\tilde{d}-1} X dr = - \sum_A \frac{q_A + 1}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_b \tilde{d}}{r^{\tilde{d}+1}}, \quad (23)$$

where  $c_b$  is another integration constant. These equations involve an unknown function  $X(r)$  and appear intractable. However,  $X(r)$  is not an independent variable but is given by (9). We now show that  $X(r)$  and  $f(r)$  can be determined from a constraint and that other functions  $u_\alpha(r)$  ( $\alpha = 0, \dots, p$ ),  $\phi(r)$  and  $B(r)$  can then be solved consistently together with the electric (magnetic) background  $E_A(r)$ .

Using the definition of  $X(r)$ , we can combine eqs. (11), (12) and (14) appropriately to derive the constraint satisfied by  $X(r)$  and  $f(r)$ :

$$\frac{X''}{X} + \left( \frac{3}{2} \frac{f'}{f} + \frac{2\tilde{d}+1}{r} \right) \frac{X'}{X} + \frac{1}{2} \frac{f''}{f} + \frac{(3\tilde{d}+1)}{2r} \frac{f'}{f} + \frac{(f-1)}{f} \frac{\tilde{d}^2}{r^2} = 0. \quad (24)$$

Note that there are terms independent of  $X$ . Since  $X$  and  $f$  can be regarded as independent functions, it is natural to set the  $X$ -independent part to 0:<sup>5</sup>

$$f'' + \frac{(3\tilde{d}+1)}{r} f' + 2(f-1) \frac{\tilde{d}^2}{r^2} = 0. \quad (25)$$

Solving this second order differential equation gives  $f(r) = (1 - \frac{\mu_1}{r^{\tilde{d}}})(1 - \frac{\mu_2}{r^{\tilde{d}}})$  with two integration constants  $\mu_1$  and  $\mu_2$ . It turns out, however, that the parameter  $\mu_2$  can be absorbed if we redefine the coordinate as  $\tilde{r}^{\tilde{d}} = r^{\tilde{d}} - \mu_2$  and  $\mu_1$  is shifted by  $\mu_2$ .<sup>6</sup> So we can simply put  $\mu_2 = 0$  without loss of generality and set

$$f(r) = 1 - \frac{\mu}{r^{\tilde{d}}}, \quad (26)$$

which characterizes the type 2 non-extreme extension. Using eq. (26) in eq. (24), we find

$$X(r) = 1 - (\nu - 1) \frac{(f^{1/2} - 1)^2}{2\sqrt{f}}, \quad (27)$$

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<sup>5</sup>There is the freedom of reparametrization of the coordinates in the metric (6). This  $f(r)$  corresponds to a choice of gauge without any loss of generality. This choice is useful to make the interpolation between the solutions of type 1 and 2 manifest.

<sup>6</sup>This shift is not a symmetry of the system, and it may appear strange that  $\mu_2$  can be absorbed by this. We have actually solved all the field equations keeping  $\mu_1$  and  $\mu_2$  and found that the parameter  $\mu_2$  could be eliminated by this shift after cancellation of various factors. For example, if we put  $f(r) = f_1(r)f_2(r)$  into eq. (24) with  $f_i(r) = 1 - \frac{\mu_i}{r^{\tilde{d}}}$  ( $i = 1, 2$ ), we get  $X = 1 - (\nu - 1) \frac{(f_1^{1/2} - f_2^{1/2})^2}{2\sqrt{f_1 f_2}}$ . After the shift, we find  $f_1(r) = (1 - \frac{\mu_1 - \mu_2}{\tilde{r}^{\tilde{d}}})f_2(r)$ , and  $f_2(r)$  drops out of  $X(r)$ , giving eq. (27). The same observation is also made for the solutions found in ref. [13].

where  $\nu$  is yet another integration constant. The choice  $\nu = 1$  reduces the solution to type 2 non-extreme case. Thus this parameter  $\nu$  introduces another direction of non-extremality. Note that the function  $X$  should contain in general two arbitrary constants, one of which is eliminated by the requirement of asymptotic flatness:  $u_\alpha(r)$  ( $\alpha = 0, \dots, p$ ),  $\phi(r)$ ,  $B(r) \rightarrow 0$  for  $r \rightarrow \infty$  requires  $X(r) \rightarrow 1$ .

Using eqs. (14), (21), (22), (23) and (27) in (13) yields

$$\begin{aligned}
& \left( \sum_A \frac{D - q_A - 3}{2(D-2)} c_A \frac{E_A}{r^{\tilde{d}+1}} - \frac{1}{2} f' X + \frac{c_0 \tilde{d}}{r^{\tilde{d}+1}} \right)^2 + \sum_{\alpha=1}^p \left( \sum_A \frac{\delta_A^{(\alpha)}}{2(D-2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\alpha \tilde{d}}{r^{\tilde{d}+1}} \right)^2 \\
& + \tilde{d} \left( - \sum_A \frac{q_A + 1}{2(D-2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{1}{r} [\nu - (\nu - 1) f^{1/2} - f X] + \frac{c_b \tilde{d}}{r^{\tilde{d}+1}} \right)^2 \\
& + \frac{1}{2} \left( - \sum_A \frac{\epsilon_A a_A}{2} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\phi \tilde{d}}{r^{\tilde{d}+1}} \right)^2 + f' X \left( \sum_A \frac{D - q_A - 3}{2(D-2)} c_A \frac{E_A}{r^{\tilde{d}+1}} - \frac{1}{2} f' X + \frac{c_0 \tilde{d}}{r^{\tilde{d}+1}} \right) \\
& - f X \left( \frac{f'}{f} + 2 \frac{X'}{X} \right) \left( - \sum_A \frac{q_A + 1}{2(D-2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{1}{r} [\nu - (\nu - 1) f^{1/2} - f X] + \frac{c_b \tilde{d}}{r^{\tilde{d}+1}} \right) \\
& + f X^2 \left[ \frac{f''}{2} + f \left( \frac{X'}{X} \right)' + \frac{\tilde{d} - 1}{2r} f' + \left( \frac{f'}{2} - \frac{f}{r} \right) \frac{X'}{X} - (f - 1) \frac{\tilde{d}}{r^2} \right] \\
& = \frac{1}{2} f X \sum_A \frac{c_A}{r^{\tilde{d}+1}} E_A'.
\end{aligned} \tag{28}$$

This equation must be valid for functions  $E_A$  of  $r$ .

With the help of eqs. (26) and (27), the  $E_A$ -independent part of eq. (28) yields a constraint condition among the constants introduced above:

$$\sum_{\alpha=0}^p c_\alpha^2 + \tilde{d} c_b^2 + \frac{1}{2} c_\phi^2 - \frac{\tilde{d} + 1}{2\tilde{d}} \left( \nu - \frac{1}{2} \right) \mu^2 = 0, \tag{29}$$

where we have redefined  $c_b$  by a constant shift ( $c_b \rightarrow c_b - \frac{\mu\nu}{2\tilde{d}}$ ). The  $E_A$ -dependent part of eq. (28), on the other hand, can be rewritten as

$$\sum_{A,B} \left[ M_{AB} \frac{c_A}{2} + \left( r^{\tilde{d}+1} f X \left( \frac{1}{E_A} \right)' + \frac{\tilde{c}_A}{E_A} \right) \delta_{AB} \right] \frac{c_B}{2} \frac{E_A E_B}{r^{2\tilde{d}+2}} = 0, \tag{30}$$

where

$$M_{AB} = \sum_{\alpha=0}^p \frac{\delta_A^{(\alpha)} \delta_B^{(\alpha)}}{(D-2)^2} + \tilde{d} \frac{(q_A + 1)(q_B + 1)}{(D-2)^2} + \frac{1}{2} \epsilon_A a_A \epsilon_B a_B, \tag{31}$$



and

$$\tilde{c}_A = 2\tilde{d} \sum_{\alpha=0}^p \frac{\delta_A^{(\alpha)}}{D-2} c_\alpha - 2\tilde{d}^2 c_b \frac{q_A+1}{D-2} - \tilde{d} \epsilon_A a_A c_\phi. \quad (32)$$

Note that for  $\nu < \frac{1}{2}$ , eq. (29) tells us that  $c_\alpha = c_b = c_\phi = \mu = 0$ , and this does not give nontrivial solution. The same is true for  $\nu = \frac{1}{2}$ . Hence we restrict ourselves to  $\nu > \frac{1}{2}$ . Since  $M_{AB}$  is constant, eq. (30) cannot be satisfied for arbitrary functions  $E_A$  of  $r$  unless the second term inside the square bracket is a constant. Substituting eqs. (26) and (27) into this differential equation, one obtains the solution

$$E_A(r) = \frac{N_A}{1 - \beta_A(1 - g^{-\alpha_A})}, \quad (33)$$

where  $\beta_A$  and  $N_A$  are integration constants, and

$$g(r) = \left| \frac{f^{1/2} - \rho}{\rho f^{1/2} - 1} \right|, \quad \alpha_A = \frac{2}{\tilde{d}\sqrt{2\nu-1}\mu} \tilde{c}_A, \quad (34)$$

where parameter  $\rho$  is defined as

$$\rho \equiv \frac{\nu - 1}{\nu + \sqrt{2\nu - 1}}. \quad (35)$$

Equation (30) has two implications if we take independent functions for the background fields  $E_A(r)$ . In this case, first putting  $A = B$  in eq. (30), we learn that

$$\frac{c_A}{2} = \frac{\tilde{c}_A(\beta_A - 1)}{N_A M_{AA}} \equiv \frac{\tilde{c}_A(\beta_A - 1)}{N_A} \frac{D-2}{\Delta_A}, \quad (36)$$

where  $\Delta_A$  is given in

$$\Delta_A = (q_A + 1)(D - q_A - 3) + \frac{1}{2} a_A^2 (D - 2). \quad (37)$$

By use of eqs. (26), (27), (33)–(37), we integrate eqs. (21)–(23) to obtain the results

$$\begin{aligned} u_0(r) &= - \sum_A \frac{D - q_A - 3}{\Delta_A} \ln H_A + \frac{2c_0}{\sqrt{2\nu-1}\mu} \ln g - \frac{1}{2} \ln f, \\ u_\alpha(r) &= - \sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \ln H_A + \frac{2c_\alpha}{\sqrt{2\nu-1}\mu} \ln g, \quad (\alpha = 1, \dots, p), \\ \phi(r) &= \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A} \ln H_A + \frac{2c_\phi}{\sqrt{2\nu-1}\mu} \ln g, \\ B(r) &= \sum_A \frac{q_A+1}{\Delta_A} \ln H_A + \frac{2c_b}{\sqrt{2\nu-1}\mu} \ln g + \frac{1}{\tilde{d}} \left( \frac{1}{2} \ln f + \ln X \right), \end{aligned} \quad (38)$$

where  $H_A(r)$  is given by

$$H_A(r) = N_A E_A^{-1} g^{\alpha_A} = [1 - \beta_A(1 - g^{-\alpha_A})] g^{\alpha_A}, \quad (39)$$

and the integration constants are fixed by the requirement that the metrics approach to 1 asymptotically.

Using eq. (38), one can write down the expression for  $S_A(r)$  as

$$S_A(r) = N_A^2 E_A^{-2} f g^{\alpha_A}. \quad (40)$$

Now, using eqs. (19) and (36), we can determine the normalization constant  $N_A$  as

$$N_A^2 = \frac{2(\beta_A - 1)}{\beta_A} \frac{(D - 2)}{\Delta_A}. \quad (41)$$

We also have

$$\sum_{\alpha=0}^p c_\alpha + \tilde{d} c_b = 0, \quad (42)$$

from the relation (9). By use of this relation,  $\tilde{c}_A$  in eq. (32) can also be written as

$$\tilde{c}_A = \tilde{d} \left( 2 \sum_{\alpha \in q_A} c_\alpha - \epsilon_A a_A c_\phi \right). \quad (43)$$

Our metric and background fields are thus finally given by, after putting all the warp factors etc. that we get by solving the Einstein equations,

$$\begin{aligned} ds_D^2 &= \prod_A H_A^{2\frac{q_A+1}{\Delta_A}} \left[ - \prod_A H_A^{-2\frac{D-2}{\Delta_A}} g^{4c_0/(\sqrt{2\nu-1}\mu)} dt^2 + \sum_{\alpha=1}^p \prod_A H_A^{-2\frac{\gamma_A^{(\alpha)}}{\Delta_A}} g^{4c_\alpha/(\sqrt{2\nu-1}\mu)} dy_\alpha^2 \right. \\ &\quad \left. + (fX^2)^{1/\tilde{d}} g^{4c_b/(\sqrt{2\nu-1}\mu)} \left( f^{-1} dr^2 + r^2 d\Omega_{\tilde{d}+1}^2 \right) \right], \\ E_A(r) &= \pm \sqrt{2 \frac{\beta_A - 1}{\beta_A} \frac{D - 2}{\Delta_A}} H_A^{-1} g^{\alpha_A}, \end{aligned} \quad (44)$$

where we have defined

$$\gamma_A^{(\alpha)} = \begin{cases} D - 2 \\ 0 \end{cases} \quad \text{for} \quad \begin{cases} y_\alpha \text{ belonging to } q_A\text{-brane} \\ \text{otherwise} \end{cases}. \quad (45)$$

The second condition following from eq. (30) is  $M_{AB} = 0$  for  $A \neq B$ . As shown in ref. [15], this leads to the intersection rules for two branes. If  $q_A$ -brane and  $q_B$ -brane intersect over  $\bar{q}$  ( $\leq q_A, q_B$ ) dimensions, this gives

$$\bar{q} = \frac{(q_A + 1)(q_B + 1)}{D - 2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B. \quad (46)$$

For eleven-dimensional supergravity, we have electric 2-branes, magnetic 5-branes and no dilaton  $a_A = 0$ . The rule (46) tells us that 2-brane can intersect with 2-brane on a point ( $\bar{q} = 0$ ) and with 5-brane over a string ( $\bar{q} = 1$ ), and 5-brane can intersect with 5-brane over 3-brane ( $\bar{q} = 3$ ), in agreement with refs. [9, 10].

The solutions (44) are the general intersecting branes which interpolate non-extreme solutions of type 1 and 2. As mentioned before, for  $\nu = 1$ , we have  $X = 1, g = f^{1/2}$  and the above solutions give generalized non-extreme solutions of type 2 with  $p+m+2$  parameters  $c_\alpha (\alpha = 0, \dots, p), c_b, c_\phi, \beta_A (A = 1, \dots, m)$  and  $\mu$  restricted by 2 constraints (29) and (42). If we further choose  $c_0 = \frac{\mu}{2}, c_b = -\frac{\mu}{2\tilde{d}}, c_\alpha = c_\phi = 0, (\alpha = 1, \dots, p)$ , they reduce to the known solutions (see for example [15]).

It appears that they no longer give non-extreme solutions of type 1 if we put  $\mu = 0$  since then the non-extreme function  $X$  in (27) becomes 1. However, we can manage to derive such solutions as follows. Consider the limit sending  $\mu$  to zero. If we keep the combination

$$\frac{\nu - 1}{8}\mu^2 \equiv r_0^{2\tilde{d}}, \quad (47)$$

finite, we get nontrivial functions

$$X(r) = 1 - \left(\frac{r_0}{r}\right)^{2\tilde{d}}; \quad g(r) = \frac{1 - (r_0/r)^{\tilde{d}}}{1 + (r_0/r)^{\tilde{d}}}. \quad (48)$$

It is then easy to see that the solutions reproduce the non-extreme ones of type 1 discussed in ref. [18].

It would be instructive to explicitly give the single brane case. The metric is

$$\begin{aligned} ds_D^2 = & H^{\frac{2(p+1)}{\Delta}} \left[ H^{-2\frac{D-2}{\Delta}} \left( -g^{4c_0/(\sqrt{2\nu-1}\mu)} dt^2 + g^{4c_u/(\sqrt{2\nu-1}\mu)} \sum_{\alpha=1}^p dy_\alpha^2 \right) \right. \\ & \left. + (fX^2)^{1/\tilde{d}} g^{4c_b/(\sqrt{2\nu-1}\mu)} \left( f^{-1} dr^2 + r^2 d\Omega_{\tilde{d}+1}^2 \right) \right], \end{aligned} \quad (49)$$

where we have set  $c_1 = c_2 = \dots = c_p \equiv c_u$ ,  $\tilde{c} = \tilde{d}[2c_0 + 2pc_u - \epsilon ac_\phi]$ , and the other quantities  $\Delta$ ,  $H(r)$  and  $\alpha$  are given in (37), (39) and (34) with the subscript  $A$  (which is irrelevant for a single brane) removed and  $q$  replaced by  $p$ , respectively. There are five independent parameters in the above single brane metric. Namely we have seven integration constants  $c_0, c_u, c_b, c_\phi, \beta, \nu$  and  $\mu$  restricted by the two constraints from eqs. (29) and (42).

Our single brane solution includes that of ref. [17] as a special case which is a four parameter solution. If we consider the limit  $\mu \rightarrow 0$  keeping eq. (47) finite, our solution (49) reduces to a single brane case with  $X(r)$  and  $g(r)$  in eq. (48), and  $\alpha = \frac{1}{2\tilde{d}r_0^{\tilde{d}}} \tilde{c}$ , with constraints

$$c_0^2 + pc_u^2 + \tilde{d}c_b^2 + \frac{1}{2}c_\phi^2 - 4\frac{\tilde{d}+1}{\tilde{d}}r_0^{2\tilde{d}} = 0, \quad (50)$$

and (42). This solution contains four independent parameters ( $c_0, c_u, c_b, c_\phi, \beta$  and  $r_0$  restricted by the two constraints). It is easy to transform our solution to the complete solution of [17] with redefinition of parameters.

To summarize, we have given very general intersecting brane solutions without assuming any restriction on the metric such as (2). The result is a general class of the brane solutions which interpolate the non-extreme solutions of type 1 and 2, which are expected to give further insight into the nonperturbative effects in string and field theories. The method we use is a simple generalization of the one in ref. [15], which can also be applied to time-dependent cases as well [21]. It is gratifying to find that the method is so useful.

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